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A fundamental equation for Supermassive Black Holes

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We developed a theoretical model able to give a common origin to the correlations between the mass M_{\bullet} of supermassive black holes and the mass, velocity dispersion, kinetic energy and momentum parameter of the corresponding host galaxies. Our model is essentially based on the transformation of the angular momentum of the interstellar material, which falls into the black hole, into the angular momentum of the radiation emitted in this process. In this framework, we predict the existence of a relation of the form $M_{\bullet} \propto R_e \sigma^3$, which is confirmed by the experimental data and can be the starting point to understand the other popular scaling laws too.

Keywords: Black hole physics; galaxies: general.

1. Introduction

At present, thanks to the improved angular resolution of modern telescopes, the experimental evidence indicates that a variety of nearby galaxies host a supermassive black hole (SMBH; $M_{\bullet} > 10^6 M_{\odot}$) at their center^{1–2}. The subsequent discovery of a large number of scaling laws, in which the mass of SMBHs correlates with the properties of the host galaxies (bulges)^{3–11}, demonstrates a link between the process of accretion of SMBHs and the formation and evolution of their galaxy.

Even if many analytical and numerical models have been proposed at the same time in order to explain the observed scaling relationships (see for example Refs. 12–17), the physical origin of these correlations, as well as the answer to the question “what is the most fundamental one?”, are still unclear and under debate^{18–19}.

Here we propose a simple theoretical model able to give a common origin for the correlations between the mass M_{\bullet} of SMBHs and the mass, velocity dispersion, kinetic energy and momentum parameter of the corresponding bulges. Starting from

a principle of conservation of the angular momentum and using, as a first approximation, the Bondi–Hoyle–Lyttleton (BHL) theory of accretion^{20–22}, we found a fundamental equation of the form $M_{\bullet} \propto R_e \sigma^3$, where R_e and σ are the bulge effective radius and effective stellar velocity dispersion, respectively. From the projections of this fundamental plane, using suitable correlations, we easily derive the other popular scaling laws. Despite the drastic hypotheses and hence the simplicity of the model, our results show an excellent agreement between its predictions and the experimental data, indicating that we have found a basic law for galaxies and their SMBHs.

2. The model

Marconi and Hunt in 2003 were the first to note that M_{\bullet} is significantly correlated both with σ , and with R_e . The conclusion of their study was that a combination of σ and R_e is necessary to drive the correlations between M_{\bullet} and other bulge properties⁶. This topic was then deeply investigated through simulations of major galaxy mergers, which defined a fundamental plane, analogous to the fundamental plane of elliptical galaxies, of the form $M_{\bullet} \propto R_e^{1/2} \sigma^3$ or $M_{\bullet} \propto M_{\star}^{1/2} \sigma^2$, and $M_{\bullet} \propto (M_{\star} \sigma^2)^{0.7}$, where M_{\star} is the bulge stellar mass^{23–24}. These scaling laws are very similar to what was really found observationally^{25–28}. Following this path we build a theoretical model able to explain all the famous relations linking the mass of SMBHs with the properties of their bulges.

Our model is mainly based on two hypotheses: the conservation of angular momentum and a suitable velocity field of the gas. Then we need to make an approximation to estimate the accretion radius of the black hole, which can be found either in a rough way or recurring to the BHL theory of accretion.

Let us consider a black hole of mass M_{\bullet} emitting radiation at rate L_{ε} , and accreting from a distribution ρ of gas, whose inward velocity is V_{in} .

First, we assume that the angular momentum is conserved in such a way that a part (M_{acc}) of the total mass of the gas contained in the galactic bulge will be captured by the black hole, converting its angular momentum into the angular momentum of the perpendicularly emitted radiation (with an effective mass M_{rad}). Of course, there are other mechanisms of conversion, transport or dissipation of angular momentum (viscosity, change in the spin of central black hole, etc.) that, in the past, have contributed with a different weight and importance to the accretion of black holes and probably are still acting now, but our drastic hypothesis focuses the attention only on the above described process. More complicated and detailed models should involve effects due to the other mechanisms that in our approximation are neglected.

An order of magnitude estimate of the angular momentum for the galactic bulge^{29–30} leads to the conservation equation:

$$M_{\text{acc}} R_e V_{\text{rot}} = c M_{\text{rad}} R_A, \quad (1)$$

where R_e is the effective radius of the bulge, V_{rot} is the mass-weighted mean rotational velocity of the gas, c is the speed of light and R_A the accretion radius of the Black hole.

Then, we suppose that the velocity of the gas in the galactic bulge is related to the effective stellar velocity dispersion σ in such a way that $V_{\text{in}} = \sigma$ (see Refs. 11 and 31) and $V_{\text{rot}} = A\sigma$, where A is a constant³². In the ideal case of the isothermal sphere, A is equal to $\sqrt{2}$.

Now we must estimate the accretion radius and as a first approximation we can use the Bondi–Hoyle–Lyttleton theory^{20,21}. In their model the rate at which the gas is accreted onto the black hole is

$$\dot{M}_{\text{acc}} = \pi R_B^2 V_{\text{in}} \rho, \quad (2)$$

where

$$R_B = 2GM_{\bullet}/V_{\text{in}}^2 \quad (3)$$

is the BHL radius and G is the gravitational constant. Of course the BHL theory is a realistic model in the case of radial accretion with low transverse velocity but it can be used as a good approximation even in presence of angular momentum (see the Appendix A). Anyway, also without considering the BHL theory, the expression of R_B in Eq. (3), with $V_{\text{in}} \simeq \sigma$, roughly corresponds to the well known gravitational radius of influence of a black hole, which we can adopt as an estimate of R_A .

Hence, we consider $R_A \simeq R_B$ (see the Appendix A), and substitute Eq. (3) in Eq. (1). Finally, recalling our hypothesis on the velocity field, we obtain the fundamental equation for supermassive black holes we were seeking:

$$M_{\bullet} = \frac{A R_e \sigma^3}{2 \varepsilon G c} \simeq 4.4 \times 10^7 \left(\frac{A}{\sqrt{2}} \right) \left(\frac{0.1}{\varepsilon} \right) \left(\frac{R_e}{\text{kpc}} \right) \left(\frac{\sigma}{200 \text{ km/s}} \right)^3, \quad (4)$$

where ε is the mass to energy conversion efficiency, which is set by the amount of rest mass energy of matter accreted onto the black hole that is extracted and radiated outward ($\varepsilon = M_{\text{rad}}/M_{\text{acc}}$).

The accreting black hole liberates energy at a rate

$$L_{\varepsilon} = \varepsilon \dot{M}_{\text{acc}} c^2 = \ell L_{\text{Edd}}. \quad (5)$$

It is commonly assumed that the accretion of matter onto a black hole releases energy at 10% efficiency so that a fixed value of $\varepsilon = 0.1$ is usually adopted³³, even if the range $0.001 \leq \varepsilon \leq 0.1$ has been recently investigated¹⁷.

This radiated luminosity is related to the Eddington limit, L_{Edd} . In particular, for $\ell = 1$ the central black hole is radiating at its Eddington limit:

$$L_{\text{Edd}} = \frac{4\pi G M c m_p}{\sigma_T} = \frac{4\pi G M c}{\kappa_{\text{Edd}}}. \quad (6)$$

Here m_p is the proton mass, σ_T the Thomson scattering cross-section of the electron, and $\kappa_{\text{Edd}} = \sigma_T/m_p$ the opacity of the fully ionized hydrogen. Substituting Eq. (2)

and Eq. (6) in Eq. (5), and recalling our two hypotheses, we can determine also the gas density at the effective radius in a form:

$$\rho = \frac{2\ell}{\kappa_{\text{Edd}} A R_{\text{e}}}, \quad (7)$$

similar to that found in Ref. 13. While in other models the gas density is given among the hypotheses, in our approach it is a consequence of the theory.

We have tested the effectiveness of our model on a sample³⁴ of 58 nearby galaxies ($z \sim 0$). By using these experimental data and the corresponding errors, we report in Table 1 the best-fitting values for the slope m and the normalization b for the linear relations used in this work. These values have been calculated by the routine FITEXY³⁵ for the relation $y = b + mx$, by minimizing the χ^2 . The estimates of the $\chi^2_{\text{red}} = \chi^2/(58 - 2)$ and the Pearson linear correlation r for each relation are also shown. Errors have been calculated by using the formula reported in Appendix A of Ref. 27. The results of the fits for the $M_{\bullet} - M_{\text{G}}$ and $M_{\bullet} - M_{\text{G}}\sigma^2$ relations are in agreement with the ones obtained in a previous paper using three different samples of data²⁸ and with the results obtained by other authors while the $M_{\bullet} - \sigma$ relation requires a more careful discussion (see Appendix B). The best-fitting line for the

Table 1. Black hole–bulge correlations and fitting parameters for the considered galaxy sample.

Relation	$b \pm \Delta b$	$m \pm \Delta m$	χ^2_{red}	ϵ_0	r
$M_{\bullet} - R_{\text{e}}\sigma_{200}^3$	8.11 ± 0.03	1.00 ± 0.02	5.00	0.39	0.87
$M_{\bullet} - M_{\text{G}}\sigma^2/c^2$	4.56 ± 0.10	0.87 ± 0.02	6.03	0.40	0.87
$M_{\bullet} - \sigma_{200}$	8.21 ± 0.02	5.83 ± 0.15	6.22	0.40	0.87
$M_{\bullet} - M_{\text{G}}\sigma/c$	0.73 ± 0.19	1.01 ± 0.03	6.61	0.42	0.87
$M_{\bullet} - M_{\text{G}}$	8.74 ± 0.03	1.21 ± 0.03	7.71	0.45	0.85
$R_{\text{e}} - \sigma_{200}$	0.12 ± 0.01	2.72 ± 0.09	10.31	0.32	0.75
$R_{\text{e}} - M_{\text{G}}\sigma^2/c^2$	-1.77 ± 0.06	0.45 ± 0.01	3.26	0.16	0.92

new relationship $M_{\bullet} - R_{\text{e}}\sigma_{200}^3$ in a log–log plane is:

$$\log_{10} M_{\bullet} = (8.11 \pm 0.03) + (1.00 \pm 0.02) \log_{10}(R_{\text{e}}\sigma_{200}^3), \quad (8)$$

where σ_{200} is the velocity dispersion in 200 km sec^{−1} units, while R_{e} is expressed in kpc. The linear relation (8) has a slope equal to the unity, which is exactly the value predicted by our model in the Eq. (4).

We remark also that this relation has the best χ^2 and r among the relations in the upper part of Table 1 involving the black hole mass. We also report the intrinsic scatter ϵ_0 , finding that the relation $M_{\bullet} - R_{\text{e}}\sigma^3$ has $\epsilon_0 = 0.39$, whereas the other ones have values of $\epsilon_0 \geq 0.40$.

Furthermore, from the normalization we can calculate the effective efficiency coefficient, which turns out to be equal to $\hat{\varepsilon} = 2\varepsilon/A = 0.048 \pm 0.003$, a value close to 0.05 estimated in Ref. 12, and to 0.06 used in the case of Schwarzschild’s metric³³.

Of course if the coefficient $\hat{\varepsilon}$ depends on R_e or σ the relation (8) will still exist but its slope will be different from the unity.

In Figure 1, we report the $M_\bullet - R_e\sigma_{200}^3$ diagram in a log-log plot (we associated a particular symbol to each galaxy according to its morphology³⁴). The best-fitting line is also shown.

We also tested Eq. (4) by using an old sample of 37 objects⁶, obtaining a slope equal to 0.90 ± 0.04 , which is in good agreement with the slope of Eq. (8). Using a subset of 27 galaxies of the same old sample, Hopkins et al.²⁵ obtained a fundamental plane $M_\bullet \propto R_e^{0.43}\sigma^{3.00}$ and Graham²⁶ $M_\bullet \propto R_e^{0.28}\sigma^{3.65}$. The difference with our result is due to a different fitting method and to the fact that these authors have considered three free parameters. With a different sample also Aller and Richstone¹⁰ studied the same three parameters fit obtaining $M_\bullet \propto R_e^{0.28}\sigma^{3.16}$.

The fact that we predict a relation of the kind $M_\bullet \propto R_e\sigma^3$ and not the Hopkins result²³ $M_\bullet \propto R_e^{0.5}\sigma^3$ does not depend on an oversimplification of our model. To obtain the latter relation one must start from different hypotheses and only increasing the number of experimental data it will be possible to distinguish the better approach.

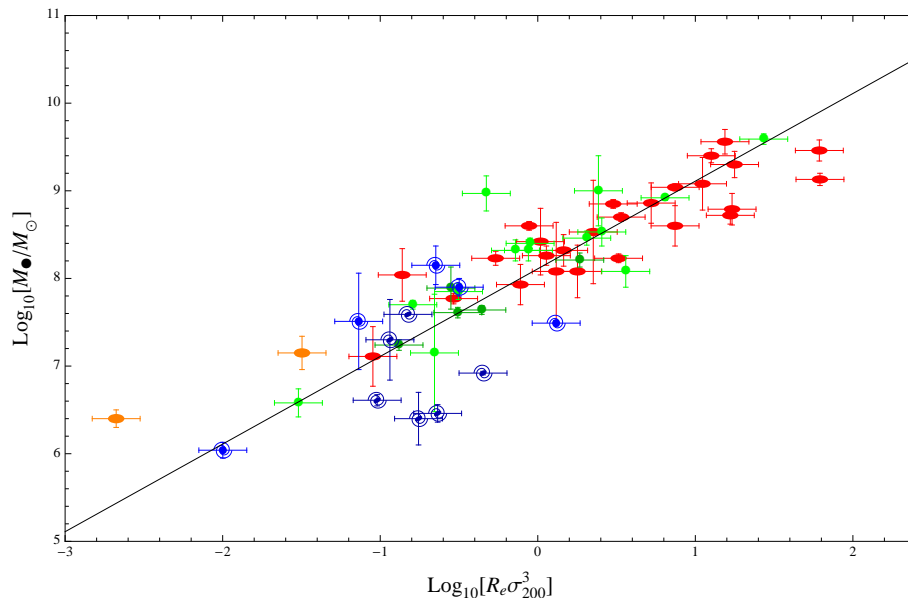


Fig. 1. The $M_\bullet - R_e\sigma_{200}^3$ relation for the galaxies of the considered data set, where σ_{200} is the bulge velocity dispersion in units of 200 km s^{-1} . The symbols represent elliptical galaxies (red ellipses), lenticular galaxies (green circles), barred lenticular galaxies (dark green circles), spiral galaxy (blue spirals), barred spiral galaxies (dark blue barred spirals), and dwarf elliptical galaxies (orange ellipses). The black line is the line of best fit for the sample of galaxies considered.

3. The origin of the other scaling relations

The dynamical masses of bulges, M_G , can be estimated by

$$M_G = \frac{k R_e \sigma^2}{G}, \quad (9)$$

where k is a model dependent dimensionless constant. We stress that the isothermal model (i.e. σ is a constant throughout the galaxy) is not a hypothesis of our framework. We make use of this approximation in order to test our model. As in Refs. 6 and 34, we use $k = 3$ to compute the “isothermal masses” of the galaxy sample considered³⁴ (in a more detailed model k is dependent on the Sersic index). Thanks to the fundamental Eq. (4), coupled with Eq. (9), in the following we derive all the most famous scaling relationships between the black hole mass and the parameters of the host galaxy.

3.1. The $M_\bullet - M_G \sigma/c$ relation

Replacing Eq. (9) in Eq. (4), we get:

$$M_\bullet = \frac{1}{\hat{\varepsilon} k} \left(\frac{M_G \sigma}{c} \right), \quad (10)$$

in optimum agreement with the corresponding relation in Table 1, where the value of the slope 1.01 is close to unity as we expected (the quantity $M_G \sigma/c$ is also known as the momentum parameter¹¹). If we impose the exponent of the momentum to be equal to 1.00, then by refitting the data we obtain a normalization $b = 0.82 \pm 0.02$, from which we derive $\hat{\varepsilon} = 0.05$.

3.2. The $M_\bullet - \sigma$ relation

Substituting the experimental relation between R_e and σ (see Table 1) in Eq. (4), we obtain:

$$M_\bullet = \frac{10^{0.12}}{\hat{\varepsilon} c G} (\sigma_{200})^{5.72} = 10^{8.23} (\sigma_{200})^{5.72}, \quad (11)$$

which is in agreement, inside the errors, with the corresponding law in Table 1:

$$M_\bullet = 10^{8.21 \pm 0.02} (\sigma_{200})^{5.83 \pm 0.15}. \quad (12)$$

3.3. The $M_\bullet - M_G \sigma^2/c^2$ relation

Using Eq. (9), Eq. (4) can be written in terms of the kinetic energy of random motions:

$$M_\bullet = \frac{1}{\hat{\varepsilon}} \left(\frac{c^2 R_e}{G k^3} \right)^{1/4} \left(\frac{M_G \sigma^2}{c^2} \right)^{3/4}. \quad (13)$$

Replacing R_e in Eq. (13) with the value $R_e = 10^{-1.77} (M_G \sigma^2/c^2)^{0.45}$, taken from the experimental relation in Table 1, we obtain:

$$M_\bullet = 10^{4.60} (M_G \sigma^2/c^2)^{0.86}, \quad (14)$$

where we have used the value of 0.048 for $\hat{\varepsilon}$. We point out the good match between the relation in Eq. (14) and the corresponding one reported in Table 1:

$$M_{\bullet} = 10^{4.56 \pm 0.10} (M_G \sigma^2 / c^2)^{0.87 \pm 0.02}. \quad (15)$$

As shown in previous papers^{27,28}, this relation has the best χ^2 compared to $M_{\bullet} - \sigma$ and $M_{\bullet} - M_G$ laws.

3.4. The $M_{\bullet} - M_G$ relation

Starting from the Eq. (10) and expressing σ in terms of M_{\bullet} (see Table 1), we obtain:

$$M_{\bullet} = 10^{-4.52} M_G^{1.21}, \quad (16)$$

in excellent agreement with the relation:

$$M_{\bullet} = 10^{-4.57 \pm 0.34} M_G^{1.21 \pm 0.03}. \quad (17)$$

4. Summary

To sum up, we have shown that the model proposed in §2 works very well since, given a consistent set of data, it perfectly predicts the slope of a new relation (Eq. 8). Moreover, the most investigated scaling relationships can be easily obtained as projections of the plane identified by the fundamental Eq. (4). Unfortunately, we cannot infer this correlation at redshift $z \gg 0$, because we are limited by the small number of observable hosts. Other mechanisms of accretion, different from the one presented in this work, may have acted in the past, and hence other fundamental equations may have ruled the first Gigayears of the life of SMBHs. Future detection of new SMBHs, especially at higher redshift, and measurements of their masses will enable us to confirm the universality of our law or if it holds just for a restricted period of the cosmic time. At this stage it is early to predict the consequences of our equation in the context of the models about the co-evolution of galaxies and black holes. First of all we must check the validity of the approach using an enlarged sample of data. Recently Sani et al. collected a new interesting (but not larger) galaxy sample³⁶ and we intend to present a complete analysis of their data in a forthcoming paper. A preliminary result is that a tight relation $M_{\bullet} - R_e \sigma^3$ exists, but its slope oscillates from 0.78 to 1.2 depending on the number of pseudobulges^a considered in the fit. This aspect requires a deeper investigation before drawing any conclusions about the evolution of supermassive black holes and galaxies.

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^aEssentially, a pseudobulge is a bulge that shows photometric and kinematic evidence for disk-like dynamics

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Appendix A

We underline that our model is not based on the BHL theory assumption. We can simply estimate the value of the accretion radius recurring to the concept of radius of influence of a black hole. Actually in our paper the BHL theory is equivalently used as another approach to have an estimate of the accretion radius even if it is well known that in presence of angular momentum the BHL approach is only a fair approximation.

In their original paper, Hoyle and Lyttleton suppose that an element of volume of the gas cloud has an initial angular momentum, but it “*loses this momentum through its constituent particles suffering collisions*” at a radius R_B ²⁰. That is, the transverse velocities of the particles, which reach the accretion line from opposite directions, annihilate reciprocally, whereas the radial component, if it is not bigger than the escape velocity (and this occurs at a distance $d \leq R_B$ from the hole), makes possible that the particles were captured by the black hole. Then they show that the collisions occur with sufficient frequency to be effective in reducing the angular momentum.

Anyway, it is possible to take into account the angular momentum, but the final result does not drastically change. In fact, in a generalization of BHL approach, the author in Ref. 22 considers the orbital velocity as the source of a lateral pressure on the accretion column. Having included this pressure term in the dynamical equations, he obtained an accretion radius R_A of the same order of magnitude of the BHL radius: $0.3 R_B < R_A < 1.75 R_B$. Both BHL and Horedt analyzed a case of pure accretion of the central object without considering the energy eventually radiated outward.

Our framework is also different compared with the spherical Bondi accretion³⁷, where a sound speed of the gas is considered, and a numerical correction factor $\alpha_B \approx 100$ (see Ref. 14), which depends on the mass profile and gas equation of state, is inserted in Eq. (2). Some authors, by analyzing the role of angular momentum, refer to that special case of spherical Bondi accretion, finding that it fails systematically to reproduce their numerical simulations³⁸. On the other hand, other authors, by using Chandra X-ray observations of several nearby elliptical galaxies, observed a tight correlation between the Bondi accretion rates (calculated from the observed gas temperature and density profiles as well as from the estimated black hole masses) and the power emerging from these systems in relativistic jets³⁹. They concluded that the Bondi formulae provide a reasonable description of the accretion process in these systems, despite the likely presence of angular momentum in the accreting gas.

We can study the effect of making the BHL theory a real assumption of our model. Let us analyze a different case (based on different hypotheses) from the one considered in section 2, assuming that the angular momentum is very small so the BHL theory is not only a fair approximation, but it works very well. Actually we can suppose that the photon is emitted by the black hole with a velocity \vec{c} in the same direction of the arriving gas particle on the accretion line. It means that \vec{c} forms an angle α with this line and with the radial component of gas velocity V_{in} and hence $A = V_{\text{rot}}/V_{\text{in}} = \tan(\alpha)$. In this case the conservation of angular momentum can be written

$$M_{\text{acc}} R_e V_{\text{rot}} = M_{\text{acc}} R_e V_{\text{in}} \tan(\alpha) = c M_{\text{rad}} R_A \sin(\alpha). \quad (18)$$

If we suppose that the accretion occurs mainly for particles with low angular momentum ($A \ll 1$), we can simplify the above equation because for $\alpha \simeq 0$ we have $\tan(\alpha) \simeq \sin(\alpha)$. In this particular case, our fundamental Eq. (4) becomes

$$M_{\bullet} = \frac{R_e \sigma^3}{2 \varepsilon G c} \quad (19)$$

and the effect to consider the BHL theory as a real hypothesis of the model reduces to the disappearing of the factor A from the fundamental Eq. (4), and all the calculations of the first part of the paper are still valid considering $\hat{\varepsilon} = 2\varepsilon$. We reported again the data of the Hu sample³⁴ in Figure 2 together with the lines representing the values of the efficiency coefficient ε predicted by Eq. (19). So, *given the SMBH mass and the effective radius and dispersion velocity of the host galaxy, the equation (19) allows a quick estimate of the efficiency of a black hole.* In this way the resulting diagram (Fig. 2) can be used to classify the black holes in terms of their efficiency provided that the host galaxies satisfy the hypotheses of the model. For example, in our sample all the galaxies have a predicted $\varepsilon < 0.25$, but the prediction could be not valid, as expected, for some pseudobulges for which the approximation of the model $A \ll 1$ does not work.

Appendix B

The slopes of the $M_{\bullet} - M_G$ and $M_{\bullet} - M_G \sigma^2$ relations remain very stable changing the sample of data or the fitting method. In Ref 28 we have used one fitting method and three different samples and the resulting slopes were (1.18, 1.22, 1.27) for the $M_{\bullet} - M_G$ relation and (0.83, 0.86, 0.91) for the $M_{\bullet} - M_G \sigma^2$ relation. Viceversa, in Ref. 27 we have used only one sample and three different fitting procedures and the slopes are still stable with values (1.15, 0.98, 1.07) and (0.80, 0.74, 0.78) respectively. These values are in agreement with the results of other authors. For instance, Hopkins et al.²⁵ find 1.05 for $M_{\bullet} - M_G$ and 0.71 for $M_{\bullet} - M_G \sigma^2$, whereas Soker et al.¹¹ find 1.07 and 0.74 respectively.

The $M_{\bullet} - \sigma$ relation has a different behavior since its slope strongly depends on the fitting method (5.06, 4.46, 4.25) in Ref. 27, and on different samples (5.26, 4.99, 5.83) in Ref. 28. In this case Hopkins et al.²⁵ find 3.96, Soker¹¹ 4.18, Ferrarese and

Ford⁴⁰ 4.86 and 5.1 and Hu⁴¹ a slope that changes from 4.01 to 5.62 depending on the considered subsample of data.

The oscillation of the slope between 3.9 and 5.9 could be due in part to the fitting method (FITEXY finds the first minimum of χ^2 and gives a slope; if one introduces the intrinsic scatter and refits until the reduced $\chi^2 = 1$, a smaller slope is generally obtained⁴²) and in part to the presence of pseudobulges in the sample. In their last paper Sani et. al.³⁶ find the same value (4.00) for the slope using three different methods, but selecting only the classical bulges. By introducing also the pseudobulges in their sample, the slope increases and the result becomes no longer stable.

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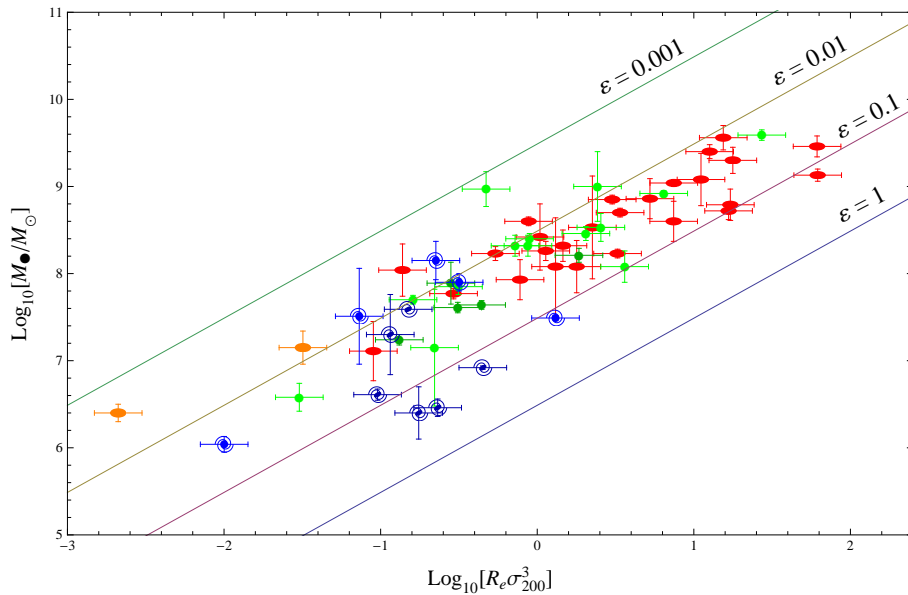


Fig. 2. The data of the considered sample in the $M_{\bullet} - R_e\sigma_{200}^3$ plane. The symbols are the same of Figure 1. Lines of constant values of the efficiency coefficient ε are shown, see Eq. (19). All the galaxies have $\varepsilon < 0.25$.

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